# Feature Clustering and Ranking

# Abstract

High dimensional remote sensing data sets typically contain redundancy amongst the features. Traditional feature selections approaches are prone to instability and selection of sub-optimal features in these circumstances. A method for ranking features that is robust to redundancy is presented. Average-linkage hierarchical clustering is used to group correlated features into clusters. Clusters are then ranked based on the value of a relevance criterion evaluated on their constituent features. Individual features can then be selected from the best clusters. Other criteria, such has computation time or measurement cost, can be considered when making this selection. These factors can have an important impact on the cost and processing time of large scale remote sensing problems. The proposed feature selection method is compared to traditional methods on a number of remote sensing data sets containing feature redundancy. Mutual information and naïve Bayes relevance criteria were evaluated in conjunction with the feature selection methods. The proposed method worked similarly well with both criteria. Stability of selected features under different data samplings was shown to be improved with the new method. Selected features produced similar or better classification accuracies, compared to traditional methods.

## Index Terms: Feature selection, hierarchical clustering, high dimensionality, correlation, redundancy

# Introduction

As the number of features increases, the amount of data required to adequately represent class distributions in the increased feature space increases exponentially. This is known as the “curse of dimensionality” (Bishop, 2003). For finite training samples, increasing the features beyond a certain point results in overtraining and a decrease in the classifier accuracy. This is called the “peaking phenomenon” (Jain et al., 2000). The peaking phenomenon makes it necessary to reduce the size of the feature set to a salient minimum in order to achieve an accurate classifier. While support vector machine (SVM) (Burges, 1998) and random forest (Breiman, 2001) classifiers have become popular, partly because of their lack of sensitivity to the peaking phenomenon (Guyon et al., 2002), reducing the number of features is also beneficial from the perspective of measurement costs and feature computation time. The last point is particularly relevant in large scale remote sensing studies involving Very High Resolution (VHR) imagery, due to the vast quantities of data requiring processing. A further motivation for reducing features to an informative minimum is the “ugly duckling theorem”, which implies that the more redundant features contained in a data set, the less separable classes become (Jain et al., 2000).

There are two basic approaches to feature set reduction: feature selection and feature extraction. In feature extraction the feature set is mapped into a new feature space of reduced dimensionality (Webb, 2002). Various criteria are used to define the dimensions of the new space such as separability and variance. These may be supervised or unsupervised measures. A disadvantage of the feature extraction approach is that it requires calculation of the full feature set. Principal Components Analysis (PCA) (Webb, 2002) is an example of a popular feature extraction method.

Feature selection involves the selection of a subset of features from the original set according to some criterion of subset performance. The number of possible subsets increases combinatorially with the size of the feature set and it is seldom practical to evaluate all possible subsets (Jain et al., 2000). A variety of search methods exist for reducing the portion of feature space searched. Of these, only the branch and bound method is globally optimal, the rest achieve reduced computation at the price of optimality. The complexity of the branch and bound method increases exponentially with the size of the feature set and so it is still computationally impractical for large feature sets (Jain et al., 2000). The fastest and most straightforward search method is simply to rank features based on their individual performance and select the best *N*. Feature ranking approaches are problematic for data sets containing feature redundancy. In these situations, correlated features are ranked similarly, resulting in sub-optimal and redundant feature sets.

More advanced search methods use greedy sequential approaches, such as forward selection and backward elimination. The forward selection (FS) approach starts with an empty feature set and proceeds in a number of steps where at each step, the feature from the remaining unselected features that improves an accuracy criterion the most is added to the model (Bishop, 2003). The backward elimination (BE) method starts with the full set of features and removes one at each step. The feature whose removal produces the best accuracy according to some criterion is eliminated from the set at each step (Bishop, 2003). BE is computationally costly and requires sufficient data to be able to train the classifier on the full feature set. Greedy search methods are more likely to find the globally optimal feature set than the feature ranking approach as they are exploring more of the search space and are less inclined to select multiple redundant features than the ranking approach.

Feature selection methods can be divided in filter, wrapper and embedded approaches. In the filter approach, generic measures of separability or importance, are used to evaluate feature subsets, while in the wrapper approach, the accuracy of a specific classifier trained on the feature subset is used as the selection criterion (Duin and Tax, 2005). A feature set that is optimal for one classifier is not necessarily optimal for another (Tolosi and Lengauer, 2011), so if using a wrapper approach, one should ideally use the same classifier to label the selected features as the criterion used for feature selection. An embedded approach is one where feature selection is incorporated into the classifier training procedure, such as with random forests (Breiman, 2001). Filter approaches have the advantage over wrapper and embedded approaches of making feature selection independent of the classifier, thus allowing for greater flexibility in the choice of classifier (Brown et al., 2012). Filter approaches are also computationally more efficient than wrapper approaches in general. This is an important consideration for large and high dimensional data sets such as those often encountered in remote sensing. For these reasons, our method adopts the filter approach.

High dimensional feature spaces typically contain feature redundancy (Cukur et al., 2015; Tolosi and Lengauer, 2011; Yu and Liu, 2004). Note that although feature correlation and redundancy are related, they are not strictly the same thing (Brown et al., 2012; Guyon and Elisseeff, 2003). While correlated features will typically be redundant, this is not always the case. Features can help improve separability when the within class correlation is stronger than the between class correlation. We use the term “redundancy” to refer to correlation of features between classes. The raw bands of aerial multi-spectral imagery often have significant spectral overlap and consequently are correlated with one another. This spectral overlap will exacerbate the redundancy amongst features derived from these raw bands (Cukur et al., 2015). In Bioinformatics, gene expression data are also recognised as typically containing substantial redundancy (Sahu and Mishra, 2011; Strobl et al., 2008; Yousef et al., 2007). A number of authors have noted difficulties in selecting features from high dimensional sets. Kononenko et al. (1997), Guyon et al. (2002), Yu & Liu (2004) and Yousef et al. (2007) noted that feature redundancy can have a negative impact on the optimality of feature selection. Feature redundancy not only leads to sub-optimal feature selection but also makes selected features unstable and sensitive to small changes in the data used for selection (Guyon and Elisseeff, 2003; Kalousis et al., 2007; Li et al., 2011; Tolosi and Lengauer, 2011).

Redundancy can be effectively dealt with using a feature extraction approach, such as PCA, but this requires computation of the full feature set. This is not practical in computationally demanding applications such as large area remote sensing of aerial imagery. A number of feature selection approaches have been developed to address the issues of stability and sub-optimality encountered in high-dimensional and redundant data. These methods consider the trade-off between feature relevance (i.e. how much information the feature contains about the class labels) and redundancy.

A means of selecting good features from redundant spaces was devised by Yousef et al. (2007). They use a k-means algorithm to produce a fixed number of clusters of correlated features. The accuracy of a SVM is found for the features of each cluster. The lowest performing clusters are eliminated, the remaining features combined, and the process repeated until a desired number of clusters is reached. A related feature selection method that finds and removes redundancy by clustering features into similar groups was presented by Mitra et al. (2002). They used a novel clustering algorithm to group correlated features based on a similarity measure they call “maximal information compression index”, which is the smallest eigenvalue of the feature covariance. Sahu & Mishra (2011) used k-means clustering to group redundant features. The best feature, according to an importance measure, is then selected from each cluster. Cukur et al. (2015) proposed a similar method, where redundant features are clustered and top ranked features selected from each cluster using an importance measure called “minimum redundancy maximum relevance” (mRMR).

A two-step procedure called the “Fast Correlation Based Filter” (FCBF), was developed by Yu & Liu (2004). It first creates a reduced set of relevant features, and then removes redundant features from this set using a search procedure based on Markov blanket filtering. A non-linear correlation measure, called symmetrical uncertainty, is used to measure both feature relevance and redundancy. Relevance is measured by how well features are correlated with class labels and redundancy is measured by how well features are correlated with each other.

A number of feature importance measures (including the FCBF) were incorporated into a common theoretical framework in Brown et al. (2012). These measures all consider both relevance and redundancy in some way. A comprehensive empirical study was used to compare the performance (in terms of stability and classifier accuracy) of these measures. The study tested the criteria in a FS approach, under varying conditions, including redundancy in high dimensional feature spaces. They concluded that joint mutual information (JMI) (Yang and Moody, 1999) provides the best feature selection performance overall.

With the exception of FCBF, the above feature selection procedures can be grouped into two categories:

1. Approaches that use some form of clustering of similar features to identify and remove redundancy. This is usually followed by a ranking of cluster relevance to choose features from the most informative clusters.
2. Approaches that use a single measure of feature importance that incorporates the trade-off between feature relevance and redundancy. A FS or simple ranking procedure can be used to select the best features.

In this paper, we propose a feature selection method for addressing the problem of high dimensional, redundant feature spaces. It follows the clustering and ranking approach of category 1 above. While the methods of Sahu & Mishra (2011) and Yousef et al. (2007) use k-means clustering, the proposed method uses hierarchical clustering. This has the advantage of producing deterministic results, not requiring prior knowledge of the number of clusters and allowing user selection of the final partitioning. The proposed method also allows consideration of computation time and measurement cost in selecting features from correlated clusters of similarly relevant features. In gene expression studies, such as DNA microarray studies, all features have similar measurement costs and computation times (Inza et al., 2004), but this is not the case for remote sensing problems, where some features might carry significantly larger computation time or measurement cost burdens than others (Blaschke, 2010). Where a group of correlated features are similarly relevant, it is beneficial to select the feature with the lowest computation time or measurement cost. Consideration of these costs can potentially result in a substantial reduction in classification computation time for this form of remote sensing problem.

Feature selection in redundant feature spaces has received attention in the field of bioinformatics, but is still largely unexplored in remote sensing applications. Of the methods reviewed, only that of Cukur et al. (2015) was applied to remotely sensed (hyperspectral) data. We compare the performance of the proposed method to popular feature selection approaches, on a number of remote sensing data sets. While many feature selection evaluations only consider classification accuracy (Cukur et al., 2015; Guyon et al., 2002; Mitra et al., 2002; Sahu and Mishra, 2011; Strobl et al., 2008; Yousef et al., 2007; Yu and Liu, 2004), we present measures of computation time, classification accuracy and stability of selected features under different data samplings.

# Methods

## Formulation

The proposed method is described as follows:

1. Perform average-linkage hierarchical clustering (Szekely and Rizzo, 2005) of the feature set using the correlation coefficient as the dissimilarity metric.
2. Select a natural number of clusters containing high correlation by visual inspection of the dendrogram.
3. Rank each cluster’s importance by finding the value of a relevance criterion for each individual feature and then finding the median of the feature relevance values in the cluster.
4. Select a single feature from each of the *N* clusters with best importance scores.

Hierarchical clustering provides a simple way of clustering the features that does not require prior knowledge of the number of clusters (Webb, 2002). The method starts with each individual feature in its own cluster and proceeds in a number of steps where the pair of clusters that minimise a criterion are merged at each step. The average linkage criterion, which is the average dissimilarity over all pairs of objects in the two clusters, was used in the proposed method. The dendrogram is a graphical representation of the clustering process. Cluster stability and strength of correlation within each cluster are the key factors to consider when choosing the number of clusters and can be visually interpreted from the dendrogram.

We investigated the performance of two different feature relevance measures: the accuracy of a naïve Bayes classifier and the mutual information (MI) between the feature and the class labels. The naïve Bayes classifier, using a histogram to model class densities, was chosen primarily because it makes no assumptions about the form of the class distributions and can thus provide a generic measure of separability. It is simple, fast and recognised as being accurate for a variety of problems (Hand and Yu, 2001). The “naïve” assumption of feature independence is of no consequence when testing individual features.

Mutual information (MI) (Brown et al., 2012) is a measure of the dependence between two random variables. Given two random variables and , with probability distributions and and joint probability distribution , the MI between and is defined as:

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| --- | --- | --- | --- |
|  |  |  | (1) |

. The mutual information between a feature and the class labels gives a useful indication of that feature’s relevance or importance (Brown et al., 2012). The probability distributions in Equation (1) are not known and are estimated using histograms.

The number of clusters to select, *N*, can be specified by the user based on the size of the training set or chosen using a grid search with the final classifier accuracy as performance measure. To avoid biased accuracy estimates, all classifier accuracy evaluation, for cluster ranking or selection of *N*, is done on unseen test data using a five-fold cross validation (Bishop, 2003). Major benefits of the cluster ranking method are its speed and that it allows hand-picking of the single features that represent each cluster. The flexibility to choose features allows the user to favour those features that are fastest to compute or perhaps to choose those features that are more readily understood. If these factors are not of importance, the highest individually ranked feature in each cluster can be chosen automatically.

## Data Sets

Five remote sensing data sets and one synthetic data set were used for comparing the proposed method against other feature selection methods. The data sets are detailed in Table 1. The “difficulty” in the last column is calculated as , as in Brown et al. (2012), where is the number of objects, the number of features and the number of classes. Smaller values mean the data is less representative of the underlying class distributions which results in more challenging feature selection and classification tasks. The Spekboom set consists of 46 spectral and textural features derived from four band multi-spectral, 0.5m pixel resolution aerial imagery. The classes represent three types of vegetation found in the Little Karoo, a semi-arid region in South Africa. It was created as part of a vegetation mapping project being conducted by the authors.

The two class synthetic data set was generated to have redundancy amongst the features. The first five features for class were generated from a normal distribution, , with mean mean and standard deviation of one (). The mean, , of each distribution, was generated from the standard normal distribution, . The same number of objects were generated for each class. To introduce redundancy, an additional five features were generated by adding normally distributed noise, , to the original five features. A further five redundant features were similarly generated, but by adding normally distributed noise, , to the original features. Finally, two spurious features, sampled from a standard normal distribution, , were added to the data set.

The Statlog Landsat and Urban Land Cover data sets were obtained from the UCI Machine Learning Repository (Lichman, 2013). The Statlog Landsat features are generated from six band multi-spectral pixel values in three by three neighbourhoods. There are six land cover classes. The features of the Urban Land Cover data are comprised of multi-scale spectral, size, shape and textural measures, derived from high resolution aerial imagery (Johnson and Xie, 2013).

Kennedy Space Centre (KSC) and Botswana are public hyperspectral data sets with vegetation and land cover classes (GIC, 2014). The Botswana data were acquired by the Hyperion sensor on board the NASA EO-1 satellite and consist of 145 bands in the 400-2500nm portion of the spectrum, at a 30m pixel resolution. The KSC data were acquired by the NASA AVIRIS (Airborne Visible/Infrared Imaging Spectrometer) sensor and consist of 176 bands in the 400-2500nm range, acquired at a spatial resolution of 18m.

Table 1 Data sets

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name | Abbrev. | Features | Objects | Classes | Difficulty |
| Spekboom | Spekboom | 46 | 57877 | 3 | 419.40 |
| Synthetic | Synthetic | 17 | 10000 | 2 | 294.12 |
| Statlog Landsat | Landsat | 36 | 3756 | 6 | 17.39 |
| Urban Land Cover | Urban | 147 | 261 | 9 | 0.20 |
| Botswana | Botswana | 145 | 1330 | 14 | 0.66 |
| Kennedy Space Centre | KSC | 176 | 1365 | 13 | 0.60 |

## Evaluation

The proposed Feature Clustering and Ranking (FCR) method was compared to a number of other established feature selection methods. These methods included the standard selection approaches of ranking, FS and BE. The standard approaches and FCR, were each evaluated with two different feature relevance criteria: MI and the naïve Bayes classification accuracy. To integrate the MI criterion into FS and BE, it is necessary to compute the MI of multiple selected features with the class labels. In this situation, the selected features are first merged into a joint variable and then the MI of the class labels with this joint variable is computed. We used histograms with 10 bins along each dimension to approximate probability densities for both the MI and naïve Bayes criteria.

In addition to the standard approaches with MI and naïve Bayes criteria, we also evaluated FS with the joint mutual information (JMI) criterion. Brown et al. (2012) compared the performance of a numerous feature selection criteria in redundant high dimensional spaces, and found the JMI criterion gave the best overall performance, in terms of classification accuracy and stability. Based on these results, FS with the JMI criterion was included to give an indication of “state of the art” performance. The evaluated methods and their criteria are detailed in Table 2. The JMI measure for feature is

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| --- | --- | --- | --- |
|  |  |  | (2) |

where are the class labels and is the set of previously selected features. JMI considers the MI between the class labels and the joint variables , which are the pairwise combinations of the candidate feature with each feature already selected. It measures how well the candidate feature complements selected features in describing the class labels.

Table 2 Method and criteria combination

|  |  |  |
| --- | --- | --- |
| Method | Criterion | Abbreviation |
| Forward selection | Joint Mutual Information | FS-JMI |
| Feature Clustering and Ranking | naïve Bayes | FCR-NaïveBC |
| Ranking | naïve Bayes | Rank-NaïveBC |
| Forward selection | naïve Bayes | FS-NaïveBC |
| Backward elimination | naïve Bayes | BE-NaïveBC |
| Feature Clustering and Ranking | Mutual information | FCR-MI |
| Ranking | Mutual information | Rank-MI |
| Forward selection | Mutual information | FS-MI |
| Backward elimination | Mutual information | BE-MI |

To quantify the stability of the selected features, we used the consistency index developed by Kuncheva (2007). If and and are subsets of the full feature set , with , and , the consistency index is

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | (2) |

It’s value lies in the range , where positive values indicate similar sets, zero indicates a random relation and negative values indicate an anti-correlation between the feature sets (Brown et al., 2012). To evaluate stability for a particular method, we select features from bootstrap samples of the data. The consistency index is found for each pairwise combination of selected features over 10 bootstraps of the data. These pairwise consistency indices are averaged to give a measure of overall stability.

As in Brown et al. (2012), a k-nearest-neighbour (k-NN) classifier (with ) was used to evaluate the accuracy of the features selected by each method. k-NN is a generic classifier that makes no assumptions about the data and requires no tuning. While other classifiers may be more accurate in particular situations, k-NN allows a relative comparison of the feature selection methods, independent of the influence of classifier tuning for specific data. For each of the feature sets found from the bootstrap samples, the k-NN accuracy was found as the average per-class accuracy from a ten-fold cross validation. For each method and data set combination, an overall accuracy was computed as the average of the bootstrap accuracies.

The number of features to select for each data set was fixed for all methods. This parameter was selected by using the accuracy of a k-NN classifier (), trained on the first *N* features selected by FS-NaïveBC, as the criterion in a grid search. A low value of *N* that achieved good accuracy was selected for each data set.

The FCR methods (FCR-MI and FCR-NaïveBC) required some specific treatment to integrate them into the evaluation. The level at which to extract the feature clusters from the hierarchy was determined by visual inspection of the dendrogram for each dataset. Levels that were both stable and strongly correlated were favoured. The chosen level was used across all bootstraps of the data set. The hierarchy levels and number of features to select for each data set are detailed in Table 3. After bootstrapping, clusters were assigned unique indices, ensuring identical clusters had the same index. The consistency index was then found using the selected cluster indices rather than feature indices. This was done to simulate hand-selection of preferred features from the best clusters for each bootstrap, while, in practice, allowing the FCR algorithm to automatically choose the top ranked feature from each cluster (for the sake of simplicity and speed). In other words, the cluster index was used to represent the index of the preferred feature that could otherwise have been selected by hand from the cluster contents. The clusters represent a reduced set of possible indices compared to the features. This reduced set in turn reduces the possibility of stability errors. It is acknowledged that this formulation of the consistency index may favour FCR, but it was regarded as a necessary modification to allow comparison of the different methods.

Table 3 Feature selection parameters

|  |  |  |
| --- | --- | --- |
| Data Set | Num Features to Select | FCR Cluster Level |
| Spekboom | 6 | 0.2 |
| Synthetic | 5 | 0.15 |
| Statlog Landsat | 4 | 0.2 |
| Urban Land Cover | 4 | 0.3 |
| Botswana | 7 | 0.03 |
| Kennedy Space Centre | 7 | 0.05 |

We followed a similar approach to that of Brown et al. (2012), for computing a single “non-dominated” ranking of the methods that considers stability and accuracy performance simultaneously. A “Pareto front” is the set of methods on which no other method can improve without degrading either the accuracy or stability. The methods in this set are called “non-dominated” (Mishra and Harit, 2010). A method was given a non-dominated rank of *N* if it was a member of the *N*th Pareto front. The average of the non-dominated ranks for each method over the 6 datasets was used to produce an overall ranking.

# Results and Discussion

The results of the stability and accuracy evaluations for each method and data set combination are shown in Figure 1 and Figure 2 respectively. The methods appear along the x axis in order of their mean stability in Figure 1, and mean accuracy in Figure 2, over the six data sets. FS-MI is the most stable overall, but has one of the poorest accuracies. Similarly, FS-NaïveBC is the most accurate overall but is the least stable. While neither FCR-NaïveBC nor FCR-MI achieve the best overall accuracy or stability, they are among the top three methods for both performance measures. The wide range of stabilities confirms the sensitivity of some methods to variations in the data. The differences in accuracy are less noticeable and the best three methods have similar accuracies. Nonetheless, there are substantial differences in accuracy between the best and worst methods. Compared to the other data sets, the stability of the Spekboom, Synthetic and Landsat data is noticeably superior. As reflected in the “difficulty” values in Table 1, these data sets are more representative of the underlying distributions and are thus less sensitive to disturbances.

The ranking methods, Rank-MI and Rank-NaïveBC both have poor accuracy performance. This is expected as these methods don’t consider feature complementarity but only measure relevance of features in isolation. The relatively poor accuracy and stability of FS-JMI was surprising in the context of the results of Brown et al. (2012), where it produced the best overall performance. Perhaps FS-JMI is more competitive when applied to higher dimensional data sets, containing hundreds or thousands of features, such as those that were used in their study. Another contributing factor to this anomaly may be that while we fixed the number of selected features to a small number, Brown et al. (2012) evaluated performance for a variable number of features and found that for some data, the superiority of the FS-JMI method only became apparent after many (more than 10) features had been selected. The FS-JMI results nevertheless provide a benchmark that helps confirm the usefulness of the FCR method for the type of data investigated in our study.

As with classifier design, there is “curse of dimensionality” problem with computing the MI of joint variables. As the number of features increases, the number of objects needed to adequately represent the feature distribution increases exponentially (Brown et al., 2012). For this reason, the MI criterion is not well suited to the BE method, which requires computation of the relevance criterion for the full feature set. This likely explains the poor performance of BE-MI in terms of both accuracy and stability.



Figure 1 Method stability per data set (methods along the x axis are ordered by their mean stability over the data sets)



Figure 2 Method accuracy per data set (methods along the x axis are ordered by their mean accuracy over the data sets)

The method execution times, summed over the six data sets, are presented in Table 4. The bulk of the software implementation was done in MatlabTM, making use of the PRTools toolbox (TU Delft, 2015). The MI and JMI criteria were however computed using the FEAST (FEAture Selection Toolbox) C implementation (Brown et al., 2012). The execution time of FCR competes well with the other methods.

Table 4 Method cumulative execution time over all data

|  |  |
| --- | --- |
| Method | Time (s) |
| Rank-MI | 5.77 |
| JMI | 6.25 |
| FCR-MI | 6.46 |
| FS-MI | 24.71 |
| FCR-NaïveBC | 122.23 |
| Rank-NaïveBC | 145.55 |
| BE-MI | 815.23 |
| FS-NaïveBC | 831.99 |
| BE-NaïveBC | 5726.67 |

Table 5 presents the non-dominant ranking of the methods, in terms of both accuracy and stability. The best ranked method overall is FCR-MI, with FCR-NaïveBC, FS-MI and FS-NaïveBC occupying the second rank position. While the FS-MI and FS-NaïveBC produce the best performance for stability and accuracy respectively, FCR-MI and FCR-NaïveBC achieve a better compromise between these two measures. The Rank-NaïveBC, Rank-MI and BE-MI methods are ranked lowest due to the known limitations of these methods.

If the clustering step were omitted, FCR-MI and FCR-NaïveBC would simplify to Rank-MI and Rank-NaïveBC respectively. FCR-MI and FCR-NaïveBC show a substantial improvement in performance compared to Rank-MI and Rank-NaïveBC which lends support to the effectiveness of the clustering step. Considering the combination of the MI and naïve Bayes criteria with each method in isolation, there is a general trend for MI to produce better stability and naïve Bayes to produce better accuracy. While FCR works well with either criterion, the results favour the use of MI as it is faster and produces a better non-dominant ranking than naïve Bayes. On the whole the evaluation study shows the FCR method to be effective at selecting accurate and stable features from high dimensional remote sensing data containing redundancy.

Table 5 Non-dominated ranking of methods by accuracy and stability

|  |  |
| --- | --- |
| Method | Rank |
| FCR-MI | 1.50 |
| FCR-NaïveBC | 1.83 |
| FS-NaïveBC | 1.83 |
| FS-MI | 1.83 |
| BE-NaïveBC | 2.83 |
| JMI | 3.00 |
| Rank-NaïveBC | 3.33 |
| Rank-MI | 3.50 |
| BE-MI | 3.50 |

# Conclusions

Feature redundancy was identified as a source of instability and inaccuracy in feature selection. A method for ranking correlated clusters of features was presented. Using hierarchical clustering, a natural number of clusters can be selected by observing the stability of correlation relationships in the data using a dendrogram. Clusters are then ranked using an importance measure, calculated as the median of a relevance criterion evaluated on each individual feature in the cluster. By selecting individual features from the best clusters, a set of informative features is found while simultaneously removing redundancy from the data. The ability to hand-pick features is beneficial as it allows other factors to be included in determining the optimal feature set such as speed of computation and physical interpretability.

The effectiveness of proposed FCR method was evaluated by comparing its accuracy, stability and execution time to a set of popular feature selection methods. A set of five remote sensing data sets and one synthetic data set, all containing redundancy, were used to compare the methods. The feature selection methods were each tested in combination with two criteria for feature relevance: the MI between the candidate feature(s) and the class labels, and the accuracy of a naïve Bayes classifier trained on the candidate feature(s). The forward selection method in combination with the JMI criterion is recognised as providing competitive performance on high dimensional problems (Brown et al., 2012), and consequently was also included in the tested methods. FS-NaïveBC provided the best accuracy performance but the worst stability performance. In a similar vein, FS-MI provided the best stability performance but the second worst accuracy performance. The FCR method performed well overall, with both naïve Bayes and MI criteria. Although FCR did not quite achieve the best performance in accuracy or stability alone, it was the highest ranked method when considering the accuracy and stability measures in combination.

While the MI criterion is attractive in its ability to describe non-linear correlations and complementarity between features (Brown et al., 2012), the curse of dimensionality limits its application in practice, particularly for high dimensional problems. Part of the motivation for the JMI and other criteria presented in Brown et al. (2012), is to circumvent this problem by formulating low dimensional approximations to MI. The naïve Bayes criterion also avoids the curse of dimensionality as it treats the features as independent. Intuitively, one may suspect that this assumption is overly crude to be useful in practice, but naïve Bayes does perform well for a variety of real world problems (Hand and Yu, 2001), including the data sets used in our study. In FCR, the features are evaluated in isolation, so dimensionality and representivity issues are less of a concern than for the FS and BE methods which evaluate combinations of features. Although FCR worked well with either criterion, the results favour the use of MI as it was faster and produced a better non-dominant ranking than naïve Bayes.

The need for user specification of clustering level is a weakness of the FCR method. This is a subjective choice and different levels can lead to different sets of selected features. No user intervention is required for the other methods considered in this study. A possible way to automate this choice would be to extract clusters from all levels in the hierarchy, select a set of features from the best clusters at each level, and then use a performance measure such as the accuracy of a k-NN classifier to choose the best set of features. The need for visual inspection of the dendrogram to make the choice of clustering level also limits the dimensionality of data that the FCR method can practically be applied to. For data sets of hundreds or thousands of features, the dendrogram would likely be too cluttered to make a sensible choice and a feature selection algorithm other than FCR may be more appropriate. It is worth noting that for problems where feature stability and user specification of preferred variables are not required, FS-NaïveBC may be a more sensible choice of feature selection method. It achieved the best accuracy results and does not require any user intervention as with FCR.

Our reasoning for the choice of dissimilarity metric for clustering, clustering algorithm and feature relevance measure was loose and other options could also prove effective. A possible limitation of the proposed method is the use of ranking for selecting the best clusters. Ranking is known to be sub-optimal in the presence of redundancy (Cover, 1974). While much of the feature redundancy will be eliminated by selecting individual features from correlated clusters, it seems likely that remnant redundancy will be present in many problems. Ideally the importance of clusters should be evaluated in the context of other clusters so that the effect of informative relationships is incorporated into the selection. This could conceivably be done by adopting a FS or BE type approach to clusters rather than features. Another aspect of our method that could benefit from further investigation is the correlation coefficient dissimilarity metric. It would be preferable to use a metric that can describe non-linear correlation, such mutual information or the symmetrical uncertainty used in Yu & Liu (2004).

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